SOME PROBLEMS IN HYDRODYNAMIC EXPLOSION THEORY

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As is well known, the model of an ideal incompressible fluid is used in M. A. Lavrent'ev's cumulative theory [1] as a model which describes the deformation and flow of certain solids (for example, metals) at high pressures. This "incompressibility approach" was developed further by Lavrent'ev himself [2] in studies in which he participated [3, 4] and in others employing his concept [5-9]. It developed that with certain additional assumptions the model of an ideal incompressible liquid could be employed to describe explosions in soils and rocks. However, practical realization of the results obtained within the framework of this model meets with difficulties both practical and principle in nature. Among the latter are the infinite energy level often found in hydrodynamic problems, but difficult to define physically. Among the former difficulties is the transition from hydrodynamic quantities to physical and mechanical ones. For example, in problems of directed explosion and uniform fractionation [3, 6], one theoretically obtains a certain pulse pressure distribution on the boundary of the disintegrating volume. Here we must first find the dependence of the momentum on explosive mass, and then replace the continuous distribution by a discrete one, placing the explosive charges in boreholes and cartridges of various diameters. Some of these problems can be dealt with empirically, while others require special study.

1. The Problem of Infinite Energy. Let an infinitely long cylindrical explosive charge of radius r_0 be located within an infinite incompressible ideal medium. The flow is described by a point source potential

$$\varphi = A \ln r + B, \quad v = \partial\varphi/\partial r = A/r, \tag{1.1}$$

where A and B are constants.

The energy integral taken over all space

$$E = \pi\rho \int_{r_0}^{\infty} v^2 r\, dr \tag{1.2}$$

becomes infinite.

We will now consider a charge of finite length $2l_0$, located perpendicular to the plane xy with ends located at the points $z = \pm l_0$. Each infinitesimally small element of this charge dl, having coordinates (0, 0, 1) creates at an arbitrary point (x, y, z) a potential corresponding to a point charge

$$d\varphi = C[(z-l)^2 + x^2 + y^2]^{-1/2} dl, \quad C = \text{const}.$$

Integrating this expression over l from $-l_0$ to $+l_0$, we obtain the potential created in the medium by the extended charge:

$$\varphi = C \ln \frac{\sqrt{(l_0 - z)^2 + r^2} + l_0 - z}{\sqrt{(l_0 + z)^2 + r^2} - l_0 - z}, \quad r^2 = x^2 + y^2.$$

In the central section, i.e., at z = 0, this expression takes on the form

$$\varphi = C \ln \frac{\sqrt{l_0^2 + r^2} + l_0}{\sqrt{l_0^2 + r^2} - l_0}.$$

The velocity in the same section

$$v = \partial\varphi/\partial r = -2Cl_0/r\sqrt{r^2 + l_0^2}$$

in the limit $l_0/r \to \infty$ coincides with the velocity of an infinite charge (1.1). In the other limiting case $r/l_0 \to \infty$ (Cl_0 = const) we obtain an expression for the velocity of a point charge $v = \text{const}/r^2$. We will consider the

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expression for the kinetic energy of a liquid within a layer of unit thickness in the central portion of the charge:

$$E = \pi\rho \int_{r_0}^{\infty} \frac{4C^2 l_0^2}{r^2 (r^2 + l_0^2)} r dr = 2\pi\rho C^2 \ln \frac{r_0^2 + l_0^2}{r_0^2}.$$

The constant C is defined from the condition

$$\varphi = -P/\rho \quad \text{at} \quad r = r_0, \; z = 0.$$

Here $P = \lim_{p \to \infty, \tau \to 0} \int_0^\tau p dt$ is the so-called pulse pressure (see [8]),

$$C = -\frac{P}{\rho} \left(\ln \frac{\sqrt{r_0^2 + l_0^2} + l_0}{\sqrt{r_0^2 + l_0^2} - l_0} \right)^{-1},$$

whence at $l_0/r_0 \gg 1$ we have, approximately,

$$C = -P/2\rho \ln \frac{2l_0}{r_0}, \quad E = \pi P^2/\rho \ln \frac{2l_0}{r_0}. \tag{1.3}$$

If we consider the flow in a ring $r_0 \leq r \leq R_0$, then instead of Eq. (1.1) we obtain

$$\varphi = -\frac{P}{\rho} \frac{\ln \frac{r}{R_0}}{\ln \frac{r_0}{R_0}}, \quad \text{such that} \quad \varphi = \begin{cases} -P/\rho, & r = r_0, \\ 0, & r = R_0. \end{cases}$$

Calculating integral (1.2) in the interval from r_0 to R_0, we obtain

$$E = (\pi P^2/\rho) \left(1/\ln \frac{R_0}{r_0} \right). \tag{1.4}$$

Comparing this to Eq. (1.3), we see that the radius of such a ring is simply equal to the charge length $2l_0$. As is evident from Eqs. (1.3), (1.4), the energy is finite if l_0/r_0 is finite. In practice the charge length $2l_0$ is always finite. Even at very long charge lengths, a limitation is imposed on $2l_0$ by the finite detonation rate. If we denote by t_c the cavity expansion time, i.e., the time at which the explosive's work is completed, then to estimate $2l_0$ we may use the relationship $2l_0 \sim Dt_c$, where D is the detonation rate.

Thus, the paradox of infinite energy is resolved from the physical viewpoint: one can use the concepts of the potential of a planar source for determination of the velocity fields and other quantities, without considering the formal divergence of integral (1.2).

2. Explosion with Ejection. Let a concentrated charge with energy E be located at a depth h from the free surface, which coincides with the plane xy. We must determine the initial velocity field developed in the ground after detonation of such a charge.

The action of the charge is replaced by a source of intensity m_0. The geometry of the initial velocity field is defined by the potential

$$\varphi = \frac{m_0}{4\pi} \left[\frac{1}{\sqrt{(z+h)^2 + r^2}} - \frac{1}{\sqrt{(z-h)^2 + r^2}} \right], \quad r^2 = x^2 + y^2.$$

The initial velocity field profile on the ground surface $v = \partial\varphi/\partial z$ at $z = 0$ has the form

$$v(r) = \frac{m_0 h}{2\pi (r^2 + h^2)^{3/2}}.$$

The edge of the ejected cone is defined by the additional condition $v(r_*) = c_*$, where c_* is the critical velocity value characteristic of the given soil. From this condition we have

$$m_0 = 2\pi c_* h^2 (1 + n^2)^{3/2}, \tag{2.1}$$

where $n = r_*/h$ is the ejection index.

We must now find the relationship between m_0 and the energy E, which is usually done from energy considerations [8]. The kinetic energy of the liquid is calculated and equated to some fraction of the charge energy E. It can easily be shown that such an approach is, in general, invalid, since the fraction of the explosive energy expended in the ejection must depend on the depth at which the charge is located. At a relatively great depth this fraction must equal zero, since an ejection does not occur. The same is true of the value m_0. Thus, m_0 must decrease with increase in h, and obviously, must increase with increase in E. From dimensionality considerations we obtain the fundamental relationship

$$m_0 = kE/\rho c_* h, \quad k = \text{const.} \tag{2.2}$$

Substituting this expression in Eq. (2.1), we obtain

$$E = K\rho c_*^2 h^3 [0.5\,(1+n^2)]^{3/2}, \quad K = \text{const.} \tag{2.3}$$

Here the function of the ejection index $f(n) = [0.5(1 + n^2)]^{3/2}$ is normalized to unity at n = 1, just as in Boreskov's expression

$$E = Kh^3 f_1(n), \; f_1(n) = 0.4 + 0.6n^3. \tag{2.4}$$

At large values of n the functions f(n) and $f_1(n)$ are of one and the same order, n^3, in contrast to the functions of n obtained previously in [8].

At high charge position depths, when a significant role is played by the force of gravity rather than the resistance of the soil, we should take $c_* \sim \sqrt{gh}$, and then from Eq. (2.3) we obtain

$$E = K\rho g h^4 [0.5(1+n^2)]^{3/2}.$$

If the depth h tends to zero, then $m_0 \to \infty$; a contact explosion corresponds to a hydrodynamic dipole with moment $M = m_0 h$, proportional to the charge energy.

3. Planar Case. As is well known, in the planar case one can introduce the complex potential $w(z) = \varphi + i\psi$. For the problem analogous to the one considered above (infinitely long and thin explosive charge), this potential has the form

$$w(z) = (m/2\pi)\ln[(z + hi)/(z - hi)]. \tag{3.1}$$

Separating the real and imaginary components, we have

$$\varphi = (m/4\pi)\ln\frac{x^2+(y+h)^2}{x^2+(y-h)^2}; \tag{3.2}$$

$$\psi = (m/2\pi)\,\mathrm{arctg}[2hx/(x^2+y^2-h^2)]. \tag{3.3}$$

On the free surface at y = 0 the velocities are directed vertically upward and have the form

$$v(x) = mh/[\pi(x^2+h^2)].$$

Equating this expression to the quantity c_*, we obtain

$$m = \pi c_* h\,(1+n^2), \quad n = x_*/h. \tag{3.4}$$

Here x_* is the coordinate on the free surface at which $v = c_*$, and n is the ejection index. The quantity m is related to the energy expended per unit length E by an expression analogous to Eq. (2.2):

$$m = kE/(\rho c_* h), \quad k = \text{const.} \tag{3.5}$$

The conclusions for the planar case are obviously the same as in the case of axial symmetry. For an infinitely deep charge the source intensity vanishes, while for a contact explosion ($h \to 0$) the charge action can be simulated by a hydrodynamic dipole with moment $M = mh \sim E$.

By substituting Eq. (3.5) in Eq. (3.4) we obtain a computation equation for explosion of a line charge with ejection:

$$E = K\rho c_*^2 h^2 (1+n^2), \quad K = \text{const.} \tag{3.6}$$

It is of interest that Eqs. (2.3) and (3.6) can be obtained from other considerations [10]. Equation (3.3), rewritten in the form

$$(x - \varkappa h)^2 + y^2 = h^2(1+\varkappa^2), \; \varkappa = \mathrm{ctg}\,(2\pi\psi/m), \tag{3.7}$$

defines a family of current lines. One of these lines, passing through the point $x_* = nh$, y = 0 can be interpreted as the boundary of the "true" ejection region (in contrast to the "visible" boundary which is finally produced by slippage or collapse of the boundaries). The parameter $\varkappa$, which determines this boundary, is related to the ejection index n by the expression $\varkappa = (n^2 - 1)/2n$. In the past a so-called hard liquid model (HLM) was proposed [2] and developed [5]. In this model the medium was considered as an ideal incompressible

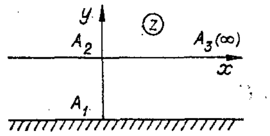

Fig. 1

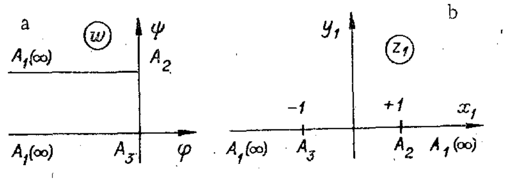

Fig. 2

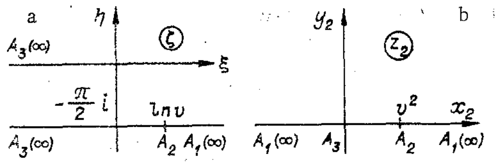

Fig. 3

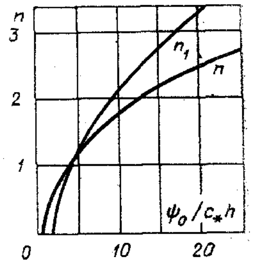

Fig. 4

liquid in a finite region surrounding the charge, limited by a flow line along which the velocity was constant and equal to c_*. The problem of explosion of a line charge with ejection was considered in the HLM in [11]. The form of the ejection region obtained therein is, generally speaking, similar to the circle described by Eq. (3.7). Since we are always concerned here with a qualitative model of the explosive action, it is obvious that in a number of cases the approach described above is possible. This variant of the incompressibility approach is sometimes called the liquid model (LM).

4. Effect of a Hard Bottom. It is well known that for an explosion on a rigid base the mechanical action of the explosion in the half space is equivalent to that of a charge of twice the energy in a full space. We must consider the situation in which a layer of soil lies upon the rigid base. In particular, we must find the effect on the dimensions of the ejection cone in comparison to an explosion at the same depth in an unbounded halfspace. We will consider this problem using the planar case and the liquid model. The flow region in the physical plane is depicted in Fig. 1. We will introduce the dimensionless variables $\bar{z} = z/h$, $\bar{w} = w/\psi_0$, where h is the layer thickness, ψ_0 is half of the liquid flow from the source, located at point A_1 (we will omit the bar in the future). We must determine the complex flow potential and the velocity field. The problem is solved by the conformal mapping method. The flow regions in the planes w and $z_1 = -\operatorname{ch} \pi w$ are shown in Fig. 2a, b. We then introduce the functions

$$\zeta = \ln (dw/dz),\ z_2 = \exp [2(\zeta + \pi i/2)].$$

The flow regions are shown in Fig. 3a, b. Mapping z_2 into z_1 with the corresponding points A_2 and A_3, we have

$$z_1 = (z_2 - v^2/2)/(v^2/2).$$

Here v is a parameter equal to the velocity at point A_2. After transformations we obtain the equation

$$dw/dz = v\operatorname{sh}(\pi w/2).$$

The solution of this equation satisfying the condition $w = i$ at $z = 0$ is

$$\ln \operatorname{th}(\pi w/4) = (\pi/2)(i - vz).$$

The parameter v is determined from the following condition: as $z \to -i$, $w \to -\infty$ $\ln \operatorname{th}(\pi w/4) \to \pi i$. Hence $v = 1$.

Transforming to dimensionless variables, after transformations we obtain

$$w(z) = (2\psi_0/\pi) \ln \frac{e^{-\pi z/2h} - i}{e^{-\pi z/2h} + i},\quad \psi_0 = m/2,\quad v = \psi_0/h. \tag{4.1}$$

We compare this expression to the potential $w_1(z)$ in the case of absence of a rigid base, as defined by Eq. (3.1)

$$w_1(z) = (\psi_0/\pi) \ln[(z + hi)/(z - hi)],\ \psi_0 = m/2,\ v_1 = 2\psi_0/\pi h. \tag{4.2}$$

The presence of the rigid base increases the velocity at the epicenter by a factor of $\pi/2$ times.

If we take $z = -hi + \varepsilon$, $|\varepsilon| \ll 1$ and expand Eqs. (4.1), (4.2) for small ε, we obtain

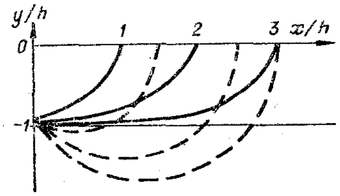

Fig. 5

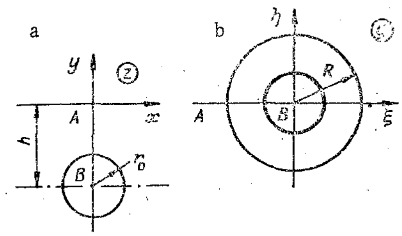

Fig. 6

$$w = (2\psi_0/\pi)\ln\varepsilon,\quad w_1 = (\psi_0/\pi)\ln\varepsilon.$$

Thus, in the close vicinity of the source the presence of the solid wall causes a doubling of the power. From Eqs. (4.1), (4.2) we define the corresponding velocity fields

$$dw/dz = -i\psi_0/\pi\,\mathrm{ch}\,\frac{\pi z}{2h},\quad dw_1/dz = -2i\psi_0 h/\pi\,(z^2+h^2),$$

and in particular, on the free surface

$$v(x) = \psi_0/\pi\,\mathrm{ch}\,\frac{\pi x}{2h},\quad v_1(x) = 2\psi_0 h/\pi\,(x^2+h^2).$$

Equating these expressions to the value of the critical velocity c_* and introducing the ejection index $n = x_*/h$, we obtain

$$n = (2/\pi)\ln\left[\psi_0/c_*h + \sqrt{(\psi_0/c_*h)^2-1}\right],\quad n_1 = \sqrt{2\psi_0/\pi c_*h - 1}.$$

The dependences of n and n_1 upon the dimensionless quantity $\psi_0/\pi c_*h$ are shown in Fig. 4. It is evident that in the presence of a rigid base the ejection index increases somewhat for $n < 1.5$. At $n > 1.5$ the rigid base decreases the dimensions of the material ejected.

From Eq. (4.1), separating the imaginary component, we have an expression for the flow function

$$\psi = (2\psi_0/\pi)\,\mathrm{arctg}\,\frac{\cos(\pi y/2h)}{\mathrm{sh}(\pi x/2h)}.$$

Hence for the current lines passing through the point $x_* = nh$, $y = 0$, we have the expression

$$\cos(\pi y/2h) = \frac{\mathrm{sh}(\pi x/2h)}{\mathrm{sh}(\pi n/2)}.$$

Figure 5 shows flow lines for n = 1, 2, 3 (curves 1-3) and the flow lines of Eq. (3.7) for n = 1.5, 2.5, 3, corresponding to an explosion without a rigid base (dashed lines).

5. Contact Explosion in the Presence of a Rigid Base. As has been shown above, within the hydrodynamic model a contact explosion is described by a dipole. If the charge is located at the point $z = 0$, then the complex potential has the form $w(z) = Mi/(2\pi z)$. The dipole axis is directed into the depths of the medium, downward in the given case, and the moment is proportional to the explosion energy: $M = \mathrm{const}\ E/\rho c_*^2$.

Now let there be a rigid base located at $z = -hi$. The velocity potential in this case can be obtained by the method of reflections from the rigid base and free surface. Upon reflection from the rigid wall the direction of the dipole axis reverses, while it remains the same upon reflection from the free surface. An infinite number of reflections produces the following expression for the potential:

$$w = (Mi/2\pi)\left[1/z + (z/2h^2)\sum_{k=1}^{\infty}(-1)^k/(k^2 + z^2/4h^2)\right].$$

Using the relationship of [12],

$$2\pi/(e^{\pi x} + e^{-\pi x}) = 1/x + 2x\sum_{k=1}^{\infty}(-1)^k/(k^2+x^2),$$

we have*

$$w(z) = (Mi/4\pi)(\mathrm{sh}\,\pi z/2h)^{-1}$$

Separating the imaginary component here, we obtain an expression for the flow function

*A similar result was obtained by A. V. Rubinskii in his candidate's dissertation, "Application of the pulsed-hydrodynamic model in the problem of explosion boundary penetration," Kazan (1981).

$$\psi = (M/4\pi)\left(\sin^2\frac{\pi y}{2h} + \mathrm{sh}^2\frac{\pi x}{2h}\right)^{-1/2}$$

and an equation for the flow line passing through the point $x_* = nh$, $y = 0$:

$$\sin^2(\pi y/2h) = \mathrm{sh}^2(n\pi/2) - \mathrm{sh}^2(\pi x/2h).$$

No graph of the flow line will be presented for this case. It is important to note the following in this case. The new interpretation of the hydrodynamic parameters characterizing the sources and dipoles proposed in Secs. 1 and 2, allows us to avoid consideration of the paradoxes related to infinite energy. In the case of a contact explosion the energy is infinite as a consequence of the singularity of z^{-1} at the origin.

Returning to the problem of the effect of a rigid base, we arrive at the conclusion that hydrodynamic theory predicts a reduction in the size of the ejected region (with the possible exception of values of $n < 1.5$). A similar result was obtained in the HLM in [8]. Table 1, containing experimental data (which was presented to the author by N. A. Trufanov), confirms this conclusion. The experiments were performed in dry sand with sections of detonating cable ≈ 1 m in length.

The rigid base was in the form of a reinforced concrete plate, located at a depth of 6 cm. S_0 denotes the area of the explosion cavity cross section in the absence of the rigid base, while S is the same value with the base.

6. Charge with Finite Cross Section. Let an infinitely long cylindrical charge of radius r_0 be located at a distance h from the free surface (Fig. 6a). On the charge surface the potential $\varphi = -P/\rho$, while on the free surface $\varphi = 0$. We must find the flow potential and velocity field.

We conformally map the flow region in the plane z into a circular ring $1 \le |\zeta| \le R$ in the plane ζ (Fig. 6b). This mapping is accomplished with the expressions

$$\zeta = R(z+bi)/(z-bi), \quad b = \sqrt{h^2 - r_0^2}, \quad R = (\sqrt{h+r_0} + \sqrt{h-r_0})/(\sqrt{h+r_0} - \sqrt{h-r_0}). \tag{6.1}$$

The boundary conditions for the complex potential $w_1(\zeta) = w[z(\zeta)]$ have the form

$$\mathrm{Re}\, w_1 = -P/\rho, \ |\zeta| = 1, \ \mathrm{Re}\, w_1 = 0, \ |\zeta| = R.$$

Under these conditions the function w_1 is uniquely defined:

$$w_1(\zeta) = (P/\rho)[(\ln\zeta)/\ln R - 1]. \tag{6.2}$$

The velocity field in the plane z is defined by the expression $dw/dz = (dw_1/d\zeta)(d\zeta/dz) = -2Pbi/\rho(z^2 + b^2)\ln R$. In particular, on the free surface at $y = 0$

$$v(x) = 2Pb/\rho(x^2 + b^2)\ln R. \tag{6.3}$$

The width of the ejected cone $2x_*$ is defined by the equality of the soil particle velocity to c_*. Denoting by $n = x_*/h$ the ejection index, from Eq. (6.3) with consideration of Eq. (6.1) we have

$$P = (1/2)\rho c_* h \ln R\left(1 + n^2 - \bar{h}^{-2}\right)\left(1 - \bar{h}^{-2}\right)^{-1/2}, \tag{6.4}$$

where $\bar{h} = h/r_0$ is the relative depth of the charge location. This expression can be changed in form if we introduce the dimensionless quantities

$$\bar{x}_* = x_*/r_0, \quad \bar{P} = P/\rho c_* r_0.$$

Then from Eq. (6.4) we have

$$\bar{x}_*^2 = (\bar{P}/\ln R)(\bar{h}^2 - 1)^{1/2} - \bar{h}^2 + 1, \quad \bar{h} \geqslant 1. \tag{6.5}$$

We will now consider the case of a contact charge. Let the center of the charge with radius r_0 be located at a distance $-h$ from the free surface. The contact line is an arc of a circle intersecting the abscissa at the points $\pm l$ (Fig. 7). The solution method is similar to the previous case: we map the semiplane with protruding segment into a semiplane [13]. On the segment $\pm l$ the value of the potential $\varphi = -P/\rho$ is specified, while outside this segment the potential is equal to zero. The solution is given by a Schwartz integral [13]. Finally we obtain

$$w(z) = [P/\rho(\pi - \alpha)]\ln\frac{z+l}{z-l}, \tag{6.6}$$

where α is the angle between the arc of the circle and the x axis. The velocity distribution on the free surface is given by the expression

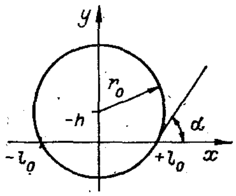

Fig. 7

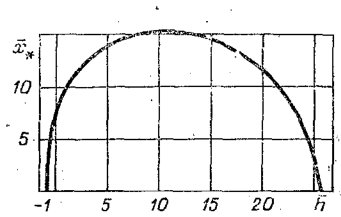

Fig. 8

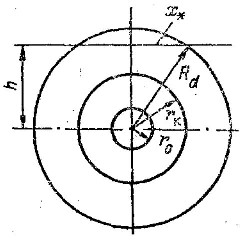

Fig. 9

$$v(x) = 2Pl/[\rho(\pi - \alpha)(x^2 - l^2)].$$

Equating this velocity to the critical velocity c_*, after transformations we obtain

$$\bar{x}_*^2 = [\bar{P}/(\pi - \alpha)](1 - \bar{h}^2)^{1/2} + 1 - \bar{h}^2, \quad 1 \geqslant \bar{h} \geqslant -1, \tag{6.7}$$

where $\bar{x}_* = x_*/r_0$; x_* is the coordinate of the ejected material edge; $\bar{h} = h/r_0$; $\bar{P} = 2P/(\rho c_* r_0)$. For a charge submerged by its own radius, Eqs. (6.2), (6.5), as well as Eqs. (6.6), (6.7) give the same results $w(z) = -2iPr_0/\rho z$, $\bar{x}_*^2 = \bar{P}$.

Thus, Eqs. (6.5), (6.7), and (6.2) give the dependence of crater width on depth of charge location over the entire range of variation of the latter. Figure 8 shows the dependence of $\bar{x}_*$ on $\bar{h}$ assuming that the quantity $\bar{P}$ is constant ($\bar{P} = 100$).

For a contact explosion in the $\bar{h}$ range from -1 to $+1$ this assumption is quite justifiable, since the action time of the detonation products is determined by the time required to expell the explosion products into the atmosphere, which changes insignificantly over this range for charges of fixed radius. In the general case, as follows from the above, for a contact explosion ($|\bar{h}| \leq 1$) $P = \text{const}\, E$.

In order to obtain the relationship between the energy E and the pulse pressure P, we substitute Eq. (6.1) in Eq. (6.2):

$$w(z) = (P/\rho \ln R) \ln [(z + bi)/(z - bi)], \quad b = \sqrt{h^2 - r_0^2}$$

and equate the expression obtained to the potential of a point source (3.1) for $h \gg r_0$, whence we obtain for the source power the expression $m = 2\pi P/(\rho \ln R)$. Thus, and from Eq. (3.5) we obtain

$$P = k(E/c_* h) \ln R, \quad k = \text{const.}$$

Substituting this expression in Eq. (6.5), we obtain an expression for the half-width of the crater x_* in terms of the linear energy density E:

$$\bar{x}_*^2 = (kE/\rho c_*^2 r_0^2)(1 - \bar{h}^{-2})^{1/2} - (\bar{h}^2 - 1). \tag{6.8}$$

The values of h and $\bar{h}$ at which $x_* = 0$ will be termed the camouflet depth and denoted by the subscript c. Taking $\bar{h} \gg 1$ in Eq. (6.8) and neglecting the unity term in comparison to $\bar{h}^2$, we have

$$\bar{x}_*^2 = \bar{h}_c^2 - 1, \quad x_*^2 = h_c^2 - h^2. \tag{6.9}$$

It is remarkable that these simple relationships can also be obtained, as noted above, from other considerations [10, 14].

We will assume that the edge of the ejection region is defined by the intersection of the free surface, i.e., the x axis, with a circle of radius R_d. By R_d we understand the radius of the destruction zone, where ejection of fragments of the medium beyond the limits of the ejection region is possible. We assume further that R_d is independent of the charge location depth. Then from Fig. 9 we can obtain Eq. (6.9) by replacement of R_d by h_c, which has obvious physical meaning. Further from similarity principles we obtain

$$h_c = R_d = K_1 r_0 = K_2 r_c = K_3 E^{1/2},$$

where r_c is the radius of the camouflet cavity; and K_1, K_2, K_3 are constants. We then obtain a computation equation of the form of Eq. (2.3). in particular,

$$q = (\pi \rho_b / K_1) h^2 (1 + n^2), \tag{6.10}$$

where ρ_b is the explosive density, q is the explosive mass per unit length of the charge.

Analogous considerations for the case of a concentrated spherical charge [10, 14] lead to expressions of

TABLE 1

h, cm	0	2	4	6
S_0, cm^2	57	132	191	203
S, cm^2	46	110	167	167

It should be noted that Eqs. (2.3), (3.6), (6.10), obtained theoretically, are approximate, as is Boreskov's empirical expression (2.4). They are valid for the cases typical of practical explosive use ($n = 1.5$–2.5 for natural soils, $n = 3$–4 for finely dispersed dry laboratory sand). Some methods of introducing semiempirical corrections were described in [14].

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